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What Fa Says About a

by Lloyd Humberstone*

Abstract

A sentence mentioning an object can be regarded as saying any one of several things about that object, without thereby being ambiguous. Some of the (logical) repercussions of this commonplace observation are recorded, and some critical discussion is provided of views which would appear to go against it.

1. Introduction to the Issue

One is inclined in informal presentations of certain logical points to speak as though, since a (declarative) sentence φ containing a name of an object a can be used to say something about that object, there must be such a thing as *what φ says about a* : what property φ says that a has, or, in another formulation, what condition it is that φ says a satisfies, or again ‘the attribute assigned to a by φ ’. (This last formulation is from Fitch (1952); note that we do not distinguish here between the name and the object, using ‘ a ’ ambiguously for either.) For example, it is often convenient to introduce logic students to the universal quantifier by telling them that a universally quantified statement says that everything has the property which is ascribed to a given object when a name for that object replaces occurrences of the variable bound by the quantifier (and the quantifier itself is dropped). If we pretend for a moment to take this seriously, then we may employ a functional notation for this ‘extraction of a ’ operation, writing, say ‘ $a\varphi$ ’ to denote the condition imposed on a by the sentence φ . For example, if φ is the sentence ‘ a is F ’ (F being some one-place predicate), then $a\varphi$ might be taken as the condition (or property) of being F . Indeed the functional notation, which originates in Fitch (1952), is legitimate if φ is literally thought of as just a sentence, since we can remove (all) the occurrences of a from φ , replacing them by occurrences of an individual variable, x say, not otherwise occurring in φ , and then form a predicate-abstract by prefixing ‘ λx ’ to the result (adapting Church’s λ -notation for functions). Thus if, as above, φ is ‘ a is F ’ then $a\varphi$ is ‘ $\lambda x.x$ is F ’. In the more orthodox language of predicate logic, we would write φ as Fa , and—in the extension of that language by λ -abstraction (for predicates)— $a\varphi$ as $\lambda x.Fx$.

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(An alternative notation, occasionally encountered, involves the use of ‘hatted’ variables, instead of λ -abstracts.)

The trouble with this syntactical construal of the ‘extraction of a ’ operation, however, is that although perfectly well-defined, it does not answer to a natural requirement we might place on the idea of taking us from a sentence containing a name to *what the sentence says about* the bearer of that name: the requirement that if ϕ and ψ are logically equivalent, then what ϕ says about anything should be what ψ says about that thing. Let us call the class of sentences logically equivalent to ϕ the *proposition* expressed by ϕ , and denote this by $[\phi]$. Then the point is that we should like to view the operation $a\backslash$ as essentially an operation on propositions, in the sense that $a\backslash\phi$, whether we continue to use that notation or instead write $a\backslash[\phi]$, should depend only on (a and) $[\phi]$.¹ That any such hope is forlorn is shown by familiar examples like the following (from Geach 1968, §43²): $[Raa] = [(\lambda x.Rxx)a] = [(\lambda x.Rax)a] = [(\lambda x.Rxa)a]$. The three λ -expressions give three different conditions which Raa can be regarded as imposing on a , or, if you prefer, three different properties which Raa attributes to a : the property of R -ing oneself, the property of being R -ed by a , and the property of R -ing a , respectively. Which of these is to be $a\backslash Raa$? Which is to be ‘what Raa says about a ’? Of course the earlier syntactic proposal does return a definite answer here, namely: it is the first of the three. (Recall that the syntactic procedure consists in replacing all occurrences of the name in question by a variable.) But this violates the requirement that $a\backslash\phi$ should depend only on $[\phi]$. Even if λ -abstraction is not present in the language, nothing can prevent us from introducing a monadic predicate G , say, stipulated to hold of an object just in case that object bears the relation R to a . Then $a\backslash Ga$, on the syntactical construal, becomes $\lambda x.Gx$, which is equivalent (by the above definition) to $\lambda x.Rxa$, while $a\backslash Raa$ becomes $\lambda x.Rxx$, so again equivalent sentences lead, on ‘extraction of a ’, to non-equivalent conditions. (Overlooking this possibility is a special case of what Geach (1968) calls the cancelling-out fallacy; we shall take a closer look at the argument just given, in Section 4 below.)

In fact, from now on we shall—apart from in incidental asides—make no further use of λ -abstraction, employing instead the open formula in one free variable to which (λ + that variable) would be attached to form the λ -abstract. We also parallel the transition, above, from sentences ϕ to propositions $[\phi]$, with a transition from open formulas such as Fx (in one free variable) to conditions $[Fx]$; where $\phi(x)$ and $\psi(x)$ are any such open formulas (of whatever internal complexity, provided that only the exhibited variable occurs free), we identify $[\phi(x)]$ and $[\psi(x)]$ when these formulas are logically equivalent in the sense that it is a logical truth that anything satisfying either satisfies the other. The point made above about $\lambda x.Rxx$, $\lambda x.Rax$, and $\lambda x.Rxa$ representing different properties can then be put by saying that not only are the λ -expressions distinct, but they—or alternatively put, the corresponding open formulas—are not even logically equivalent; in the current notation and terminology, $[Rxx]$, $[Rax]$, $[Rxa]$, are all distinct conditions.

¹ The nature of the dependence on a is obscured by our ambiguous use of ‘ a ’ as a name and for the bearer of that name; since only referentially transparent contexts will be considered, this never matters.

² Of course the point made here by Geach is an illustration of Frege’s idea of the multiple dissectability of a single unambiguous sentence; a convenient discussion is provided by Rumfitt (1994, esp. §1), which also gives references to the relevant writings of Frege, Geach and Dummett on this idea. (*Warning*: Rumfitt uses the term ‘proposition’ for a sentence understood in a particular way, rather than for the way in question.)

Now to the extent that there is a pre-theoretical notion of what a given sentence says about an object, this is the result of applying an operation, the extraction function ($a\backslash$, as it might be), whose arguments are propositions and whose values are conditions. In the above notation, we represented the arguments and values as sentences and open formulas (or the corresponding λ -expressions) respectively, and noted that equivalent sentences should yield, when this operation is applied, the same open formula: but unless an invidious selection is made of a single representative such formula, it is better to think of the *values*, as well as the arguments, as being equivalence-classes of the corresponding linguistic expressions, and thus, in the case of the values, as conditions.

These equivalence-classes may strike some readers as too large, as individuating propositions (and conditions) too coarsely. Such readers may treat what follows as an exploration of the consequences of this individuation policy (whether or not they are in sympathy with it): it certainly does not purport—numerous skirmishes with opposing ‘finer-grained’ policies in what follows notwithstanding—to be a justification of the policy. (In fact, it will not be logical equivalence but *a priori* equivalence that we emphasize below.)

We close this section by making explicit the notational conventions with which we are operating. The language under discussion is an arbitrary (interpreted) order language with identity, having in its vocabulary at least the monadic predicate letters F, G , the dyadic predicate letters R, S , and the individual constants a, b ; φ, ψ are schematic symbols for closed formulas (unless otherwise stated), with $\varphi(x), \psi(x)$ being open formulas in one free variable (as exhibited); the result of replacing each occurrence of the free variable x by the individual constant a in $\varphi(x)$ is denoted by $\varphi(x \mapsto a)$. We will use this last notation below, preferring it to slash substitution notations $\varphi(a/x)$ which are confusing because different authors have different conventions as to which of the terms flanking the slash is to be replaced by the other.

2. The Things *Fa* Says About *a*

The example of extracting a from Raa to obtain several distinct conditions, any one of which this sentence can be regarded as demanding (for its truth) that a should satisfy, suggests that we cannot take ‘What Raa says about a ’ as picking out a unique condition, but rather a *class* of conditions, for a to satisfy any one of which is both necessary and sufficient for the truth of Raa . We take ‘what’ as meaning, not ‘the (one) thing’, but ‘the (various) things’ the sentence says about the object.³ Let us adopt the notation $Cond(a, [\varphi])$ for the set of all conditions any one of which may be regarded as being imposed on a by the sentence φ (by the proposition $[\varphi]$). The precise definition can be given thus: $[\psi(x)] \in Cond(a, [\varphi])$ if and only if $\psi(x \mapsto a)$ is logically equivalent to φ . Thus amongst the elements of $Cond(a, [Raa])$ are the three conditions $[Rxx]$, $[Rax]$, and $[Rxa]$. The provisional notation $a\backslash\varphi$ of the preceding section purported to pick out the

³ I must caution against one possible misunderstanding of the plural reading of ‘what’ in ‘what a sentence says about an object’. This is not to be taken in any sense that would allow us to count, for instance, $Fa \wedge Ga$ as saying two things about a , given by the conditions $[Fx]$ and $[Gx]$. That would be to overlook the ‘and sufficient’ part of the characterization of the preceding sentence of the text.

sole element of $Cond(a, [\varphi])$, but as we saw there and have just recalled with the case of $\varphi = Raa$, the uniqueness presupposition involved is not in general satisfied.

The failure of that presupposition is not specific to the dyadic case, however, even restricting ourselves to cases in which φ is atomic. Amongst the elements of $Cond(a, [Fa])$, for example, are to be found the distinct conditions $[Fx]$ and $[x = a \leftrightarrow Fx]$. The first of these corresponds to what in our opening section was called the syntactic construal of $a \setminus Fa$. It turns out to occupy no specially distinguished role amongst the various elements of $Cond(a, [Fa])$; note, for example, that neither $[Fx]$ nor $[x = a \leftrightarrow Fx]$ logically implies the other: so $[Fx]$ is neither the logically weakest nor the logically strongest element of $Cond(a, [Fa])$. In the following section, we shall identify these weakest and strongest elements, but before getting to that a few words need to be said about a contrary impression, to the effect that $[Fx]$ does indeed occupy a distinguished role amongst the conditions the proposition $[Fa]$ imposes on a , which may be gleaned from Fitch (1952). Actually Fitch is discussing the case of propositions such as $[Raa]$, and he makes the same observation that was (later) made by Geach *à propos* of these cases, taking as his example $[c = c]$.⁴ In his own words and notation (p.94f.):

If (---c---) is any proposition mentioning c two or more times, then there are at least three different attributes that (---c---) assigns to c . (...) In other words $[c = c]$ assigns to c each of the attributes ($w \setminus [w = c]$), ($w \setminus [c = w]$), and ($w \setminus [w = w]$). But only one of the attributes assigned to c by $[c = c]$ is an attribute that does not mention c , and this we call the *principal attribute* assigned to c by $[c = c]$, and we refer to it as ($c \setminus [c = c]$). It is the same attribute as ($w \setminus [w = w]$). In general, if (...x...) is any proposition mentioning x one or more times, even if it is a proposition not mentioning x at all, we assume that there is a principal attribute ($x \setminus (...x...)$) that (...x...) assigns to x .

The opening and closing sentences here, with their talk of a proposition's mentioning an object a certain number of times, suggest that Fitch is simply confusing propositions with sentences, despite his own explicit acknowledgment of the distinction on p.6 of Fitch (1952). Likewise at the level of predicates and conditions (or attributes, to use Fitch's term), in view of the talk of attributes 'mentioning' individuals.⁵ Let us assume,

⁴ This is not as good an example, for the policy of individuating conditions here employed, since $[x = c]$ and $[c = x]$ are the same condition whereas $[Rxa]$ and $[Rax]$, for R some (non-logical) two-place predicate symbol, are distinct conditions. The identity example has also been used by Wiggins (1976) apparently to make points of the kind opposed in the present discussion; on p.231 we read: "He who says ' $a = a$ ' predicates of a what only a can have, the one-place property $\lambda x(x = a)$. Or if you will he predicates of the couple $[a, a]$ the two place relation ($\lambda x \lambda y(x = y)$). But, whichever of these we take, to ascribe it to its subject is not the same as to ascribe to a the one-place property which is ascribed by a man who says that a is identical to itself or $\lambda x(x = x)[a]$." There are two ways of taking the talk of property-ascriptions as same or different, corresponding roughly to a process/product ambiguity. To ascribe the property of being identical with a to a is not the same as to ascribe the property of being identical with itself to a if difference in properties suffices for difference in ascriptions; but if the net effect of the two ascriptions is the same, we may well want to say that in the 'product' sense, there is only ascription. An analogy: what if someone were to say (in the style of Wiggins): "to apply the squaring function to the number 2 is not the same as to apply the doubling function to the number 2"?

⁵ On p.99, Fitch does say that the sense in which an attribute mentions something is different from that in which a predicate abstract mentions something, adding "but it is convenient to employ both kinds of mentioning"; but nothing is said about how the two are related. Readers interested in the philosophical and

more charitably, that Fitch is not making any such use-mention confusions, but is instead operating with notions of proposition and attribute which are more finely individuated than the corresponding notions here: there can be, for example, distinct but logically equivalent propositions. (On p.32 of Fitch (1952), we read that “The proposition [$p \ \& \ q$] is not regarded as being the same proposition as [$q \ \& \ p$], though it is clearly the case that [$p \ \& \ q$] implies and is implied by [$q \ \& \ p$]”.⁶ The need for charity on the use-mention issue is evident from Fitch’s remarks in the same work (p.10) about compounding propositions to make ‘larger’ propositions, and indeed from the passage just cited, since Fitch’s use of square brackets is not that introduced in Section 1 above, but rather the more familiar sentence punctuation use.) Frege and Russell, for example, both held theories of propositions (or thoughts, in the case of Frege) which were perhaps sufficiently ‘syntactic’ for the idea that an object might figure in them, either by proxy (as a ‘mode of presentation’ thereof) or directly, some given number of times. Given that this is not how we construe propositions here, the interesting question for us is whether, forsaking such quasi-syntactic conceptions of the proposition, anything remains of the idea of the principal attribute assigned to an object by a proposition. We return to this question in Section 4. In Section 3, as already remarked, we shall be concerned, *inter alia*, to show that such attributes occupy no distinguished role amongst the various elements of $Cond(a, [Fa])$, if what is sought is a distinction in terms of logical strength (or weakness). We close the present section with an illustration of a more recent quasi-syntactic suggestion which bears on the example of $[Raa]$, after a remark on the word ‘about’ in such expressions as ‘what *Fa* says about *a*’.

There is a more or less traditional philosophical issue (at least since Goodman (1961)) as to when a sentence is about this or that in the sense of having this or that as its subject matter. Progress on this issue was made more recently in Lewis (1988), which shows how, for such purposes, we should conceptualize subject matters, in addition distinguishing a sentence’s being entirely about a subject matter from its being partly about a subject matter, in numerous different senses of ‘partly about’. Now the only point that needs to be made in the present context is that although we use—because of their familiarity—phrases such as ‘what the sentence ϕ says about *a*’, this usage is in no way connected with the aboutness issue just alluded to, and it would be just as good to use terminology such as ‘what ϕ says of *a*’; indeed, it would arguably be preferable for purposes of avoiding confusion. Such confusion might take the following form. An account is provided of what ϕ says about (‘of’) *a*—for example, the present suggested account in terms of $Cond(a, [\phi])$ —and this is then spliced into a proposal in the theory of aboutness: ϕ is in some sense about *a* when there is something that ϕ says about (‘of’) *a*, i.e., when $Cond(a, [\phi]) \neq \emptyset$. To see how unhelpful this move would be, recall that the heart of the traditional philosophical problem of aboutness is to provide a reasoned explication of aboutness which does not have the consequence that every sentence is about everything. Taking that as a constraint on the enterprise, the proposal currently

historical background behind those aspects of Fitch’s logical work we are touching on here will find pertinent discussions in Fitch (1971) and Pelham and Urquhart (1994).

⁶ This example is particularly convenient, since it shows that the present point does not depend at all on the fact that Fitch favours a logic weaker than the classical logic we are assuming, according to which accordingly there are pairs of sentences which do not express the same proposition on the ‘same proposition expressed by logically equivalent sentences’ individuation procedure, yet which pairs do express the same proposition by this criterion when classical logic is employed. The example chosen shows that Fitch is not employing this individuation criterion even relative to his own weaker logic.

under consideration fails dismally. For suppose that φ is not (in the sense relevant to the enterprise) about a —as must, we have just presumed, be the case for some choice of φ and a ; from our definition of $Cond(a, [\varphi])$, we can see that this set is, contrary to the proposal, far from empty: it contains, for instance, the conditions $[a = x \wedge \varphi]$, $[a = x \rightarrow \varphi]$, $[x = x \wedge \varphi]$, $[x = x \rightarrow \varphi]$, and many further conditions (which will be suggested by a perusal of Figures 1–3 in the following section, in the terminology of which the last two conditions listed here have the interesting property of being ‘possessor-independent’). Moral: nothing in our discussion should be taken as a contribution to the issue of (‘subject-matter’) aboutness. Nor—we should add especially for the sake of those familiar with Lewis’ work—is there any connection with the related issue of intrinsic properties: when people under the spell of what we shall be calling the doctrine of principal attributes single out the property of being famous as especially significant amongst the properties (or of being the one property) attributed to Mozart by the proposition that Mozart is famous, it does not bother them that the property in question is not (on any reasonable explication of the notion) an intrinsic property.

Next, the matter of quasi-syntactic or ‘fine-grained’ individuation policies for propositions. On pp.416f. of Salmon (1986) we read:

In fact there are compelling reasons to distinguish propositions of the form x bears R to x from the proposition x bears R to itself. One sort of consideration is the following: We must distinguish between the reflexive property of exceeding oneself in weight and the simple relational property of exceeding Venus in weight. (...) Now the sentence ‘Hesperus outweighs itself’ seems to ascribe to Hesperus, i.e., Venus, the impossible property of weighing more than oneself rather than the simple relational property of weighing more than Venus. It seems to say about Venus what ‘Mars outweighs itself’ says about Mars – that it has the reflexive property of exceeding oneself in weight – and not what ‘Mars outweighs Venus’ says about Mars.

Salmon’s motive in distinguishing such propositions as the proposition that a bears R to a (a proposition ‘of the form’ x bears R to x , as he puts it) from the proposition that a bears R to itself, is to block the untoward results of substituting co-referential names in the that-clauses of propositional attitude ascriptions on the assumption that such ascriptions relate the attitude-holder to a proposition whose identity is itself unaffected by such substitutions. That motive surfaces in the passage quoted only indirectly, in the peculiar and distracting ‘Hesperus’/‘Venus’ oscillation; Salmon’s intent is clearly to put forward considerations independent of issues of the semantics of attitude-ascriptions so that the distinction so motivated can simply be applied in that quarter. Thus we have a proposal according to which there are two propositions, the non-reflexive proposition that Venus outweighs Venus, and the reflexive proposition that Venus outweighs itself. Once we see that these are two propositions we see how it is possible for someone to believe the first—perhaps under the sentential incarnation ‘Venus outweighs Hesperus’—without believing the second, to believe which would, in view of its *a priori* falsity, require more than mere ignorance of astronomy. At first blush, this looks like an unpromising proposal, because the *a priori* knowability of the fact that the two propositions concerned have the same truth-value is not challenged by Salmon, so that an *a priori* and not just an *a posteriori* error would after all have to be committed by

someone who believed the first (non-reflexive) proposition while disbelieving the second (reflexive) proposition.⁷ But we do not need to examine the proposal in depth for present purposes: the quoted passage purports, after all, to adduce independent evidence for the distinction in question.

Salmon is of course correct to say that we must “distinguish between the reflexive property of exceeding oneself in weight and the simple relational property of exceeding Venus in weight”, since the properties concerned are not even co-extensive. But the idea that this difference results in a difference in propositions ascribing the properties to a given object seems flatly to ignore the possibility that there is no one preferred condition ascribed to an object by a proposition. To write that ‘Venus outweighs itself’ seems “to say about Venus what ‘Mars outweighs itself’ says about Mars (...) and not what ‘Mars outweighs Venus’ says about Mars” is precisely to succumb to the idea that there is one thing deserving to be called “what ‘Mars outweighs itself’ says about Mars”. No-one who had not already rejected the Fregean doctrine of multiple dissection⁸ could even follow such an argument because of the failed presupposition of these purportedly singular *what*-clauses.

3. The Lattice of Conditions

Before getting to the topic suggested by the title of this section, we need some logical preliminaries, and we begin by establishing notation and terminology. Some elementary observations, culminating in a simple Theorem, are then made which bear on the topic, in effect enabling us to isolate the top and bottom elements of $Cond(a,[Fa])$ when this set is construed as (the universe of) a lattice in a natural way. The remainder of the section looks at what lies between these bounds.

⁷ The argument just given cannot be thought of as an *ad hominem* argument against Salmon, however, since he thinks an *a priori* error has been committed already by anyone who believes that Hesperus outweighs Venus (or for that matter, that Hesperus is not identical with Venus). Salmon (1992) stresses that in the earlier paper he was simply presenting, and not actually espousing, the theory of reflexive propositions under discussion here; alternatively accounts, he continues to feel, leave something out: “My general dissatisfaction with the Simple Anaphor Theory stems from the fact that it leaves out the element I call reflexivity that seems present in ...” (Salmon 1992, p.54.) My own reaction to this is that we should not regard ourselves as obliged to move from finding a common pattern in several representations to positing a common element in those representations.

⁸ I say ‘multiple dissection’ rather than, as might be more natural, ‘multiple decomposition’, simply to avoid entanglement with issues raised the distinction between analysis and decomposition proposed in Dummett (1981), Chapter 15. Since we have been considering reflexive pronouns – a phenomenon present in natural languages though not in the official formal language we are focussing on in this paper – this is a suitable point at which to mention that the multiple dissectability gives rise, in conjunction with several other such anaphoric devices, to natural language ambiguities, as in the English ‘Bill loves his wife and so does Harry’; in spite of its vintage, there is a good discussion of this topic in Dahl (1973), from which that example is taken. A particularly interesting more recent discussion, Dalrymple *et al.* (1991), constrains the search for possible meanings for such elliptical constructions with the aid of a primary occurrence of an expression in a representation of the sentence – here ‘Bill love’s Bill’s wife’, with the primary occurrence underlined – in the light of which the ellipsis is to be interpreted. The constraint is that this occurrence must be replaced by the abstracting variable in the search – which still allows for more than one result – as to what ‘so does Harry’ can be interpreted as saying about Harry. The question of which occurrences are to be treated as primary is left open by the authors, as part of the unsolved explication of the syntactic notion of parallel structure.

An interpretation or model \mathbf{M} settles the extensions $|F|^{\mathbf{M}}$ of predicate symbols such as (in this illustration) the monadic F , as well as of individual constants ($|a|^{\mathbf{M}}$, etc.), while an assignment function s does so for the individual variables ($|x|^{\mathbf{M},s}$, etc.). We write $\mathbf{M} \models \varphi$ (for φ a closed formula) to mean that φ is true in \mathbf{M} , and $\mathbf{M} \models^s \varphi$ to mean that the (open or closed) formula φ is satisfied in \mathbf{M} by the assignment s . We presume the usual inductive definition of the latter notion, and of the former in terms of it. Lemma 1 is an immediate consequence of this definition:

LEMMA 1. *For any model \mathbf{M} , if $|x|^{\mathbf{M},s} = |a|^{\mathbf{M}}$ then $\mathbf{M} \models^s \varphi(x)$ iff $\mathbf{M} \models \varphi(x \mapsto a)$.*

Where $\varphi_1, \dots, \varphi_n$ and ψ are (open or closed) formulas, we write $\varphi_1, \dots, \varphi_n \vdash \psi$ to mean that for any model \mathbf{M} and any assignment s of elements of the domain of \mathbf{M} to individual variables, if $\mathbf{M} \models^s \varphi_1$, and ... and $\mathbf{M} \models^s \varphi_n$ then $\mathbf{M} \models^s \psi$. When $\varphi \vdash \psi$ and $\psi \vdash \varphi$, we write $\varphi \dashv\vdash \psi$. (In fact, only φ and ψ with at most one free variable will be considered.)

The ‘if’ and ‘only if’ parts of the consequent of Lemma 1 imply Lemmas 2 and 3 respectively:

LEMMA 2. *If $Fa \vdash \varphi(x \mapsto a)$ then $Fx, x = a \vdash \varphi(x)$.*

LEMMA 3. *If $\varphi(x \mapsto a) \vdash Fa$ then $\varphi(x), x = a \vdash Fx$.*

THEOREM. *For any open formula $\varphi(x)$ in the free variable x , $\varphi(x \mapsto a) \dashv\vdash Fa$ if and only if $x = a \wedge Fx \vdash \varphi(x)$ and $\varphi(x) \vdash x = a \rightarrow Fx$.*

Proof. ‘If’: Immediate on substituting a for x .

‘Only if’: Suppose that $\varphi(x \mapsto a) \dashv\vdash Fa$. Then Lemma 2 and the ‘ \dashv ’ half of the supposition give the conclusion that $x = a \wedge Fx \vdash \varphi(x)$, while Lemma 3 and the ‘ \vdash ’ half of the supposition give $\varphi(x) \vdash x = a \rightarrow Fx$.

To elaborate on what the Theorem tells us in terms of the apparatus of conditions and propositions of the preceding section, we recall (using the notation of the present section) the key definition, of $Cond(a, [\varphi])$, the set of conditions any one of which the proposition that φ can be thought of as imposing on a :

Def. *Cond.* $[\psi(x)] \in Cond(a, [\varphi])$ iff $\psi(x \mapsto a) \dashv\vdash \varphi$.

The conditions (in some fixed individual variable) form a boolean algebra in a familiar way; the elements of $Cond(a, [\varphi])$ form a lattice since, if $[\psi_1(x)]$, $[\psi_2(x)]$, both satisfy Def. *Cond.* for the same φ , then so do their join, $[\psi_1(x)] \vee [\psi_2(x)] = [\psi_1(x) \vee \psi_2(x)]$, and meet, $[\psi_1(x)] \wedge [\psi_2(x)] = [\psi_1(x) \wedge \psi_2(x)]$. (We use the same symbols for the lattice operations as for the corresponding connectives, as is common in discussions of Lindenbaum–Tarski algebras; note that negation does not similarly give rise to a complementation operation on the lattice since the would-be complement $\neg[\psi_1(x)]$ ($= [\neg\psi_1(x)]$) of an element of $Cond(a, [\varphi])$ does not, by contrast, belong to $Cond(a, [\varphi])$.) (For more on

complementation, see the remarks following Figure 3 below.) The implicit partial ordering \leq (w.r.t. which the above joins and meets are respectively least upper and greatest lower bounds) is given by: $[\varphi(x)] \leq [\psi(x)]$ if and only if $\varphi(x) \vdash \psi(x)$, so the bounds of the lattice—the top and bottom elements—are respectively the logically weakest and logically strongest conditions in $Cond(a, [\varphi])$; when we take φ as Fa , the Theorem above tells us that the top element is $[x = a \rightarrow Fx]$ and the bottom element is $[x = a \wedge Fx]$. These bounds turn out to be the join and the meet, respectively, of the two conditions $[Fx]$, $[x = a \leftrightarrow Fx]$, whose implicational incomparability was mentioned in the preceding section; we sketch this in a little Hasse diagram:

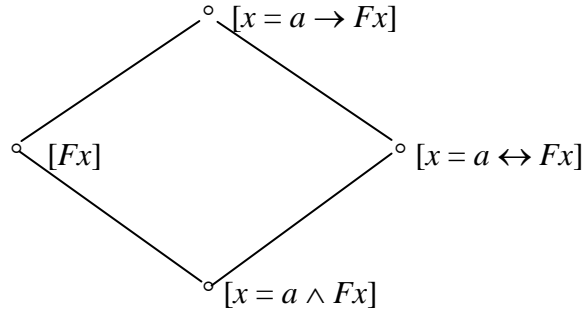


Figure 1

A diagram analogous to Figure 1 for the case in which φ is taken as Raa would of necessity be more complicated since, to focus on just one entry in Figure 1, in place of $[Fx]$, we should have to consider all of $[Rax], [Rxa]$, and $[Rxx]$, and then $[Rax \vee Rxa]$ and all the other meets and joins of these elements with each other and with the remaining conditions.

In Figure 1 itself, we have considered only conditions $[\psi(x)]$ determined by formulas $\psi(x)$ into which no non-logical vocabulary foreign to the chosen φ (i.e., Fa) intrudes; but by the Theorem above, any open formula (in x) determining a condition in $Cond(a, [Fa])$ is intermediate in logical strength between those determining the bottom and top elements of this lattice. One such implicationally intermediate formula involving extraneous vocabulary, for example, would be $(x = a \wedge Fx) \vee (x \neq a \wedge Gx)$. Thus, alongside the elements of $Cond(a, [Fa])$ listed in Figure 1, would have to be reckoned, *inter alia*, the condition $[(x = a \wedge Fx) \vee (x \neq a \wedge Gx)]$. However, we shall restrict attention here to the conditions $[\psi(x)]$, for $\psi(x)$ constructed with the aid of F, a , and the logical devices. Although, as just remarked, Figure 1 depicts only such conditions, it by no means depicts all of them. The condition $[x = x \wedge Fa]$ has been conspicuously omitted. This corresponds to what has variously been called an ‘irrelevant predication’ and a ‘possessor-independent property’, and would be expressed in (sufficiently liberal) lambda-calculus notation by means of $\lambda x.Fa$: however, as we are avoiding any such index variables (as the ‘ x ’ in ‘ λx ’ or ‘ x ’), we prefer to make the variable explicit by ‘dummying in’ the vacuous conjunct ‘ $x = x$ ’.⁹ When we take the join of this condition

⁹ On ‘dummying in’, see Dunn (1987), which is the source of the irrelevant predication idea; possessor-independent properties—those guaranteed to be possessed either by all objects or by none—are isolated under that name in Humberstone (1996).

with $[Fx]$, however, we no longer need to make special provision for the variable to occur, so we write this join in the diagram below as $[Fx \vee Fa]$. Once we add the new condition, $[x = x \wedge Fa]$, to Figure 1, and consider all meets and joins arising, we obtain the nine-element distributive lattice of Figure 2. The labelling of the conditions is of course somewhat arbitrary. For example, the top and bottom elements (which are of course the same bounds as appeared in Figure 1) could equally well have been denoted by $[x = a \rightarrow Fa]$ and $[x = a \wedge Fa]$, representations more reminiscent of the possessor-independence feature just commented on (though the conditions represented do not themselves have this feature):

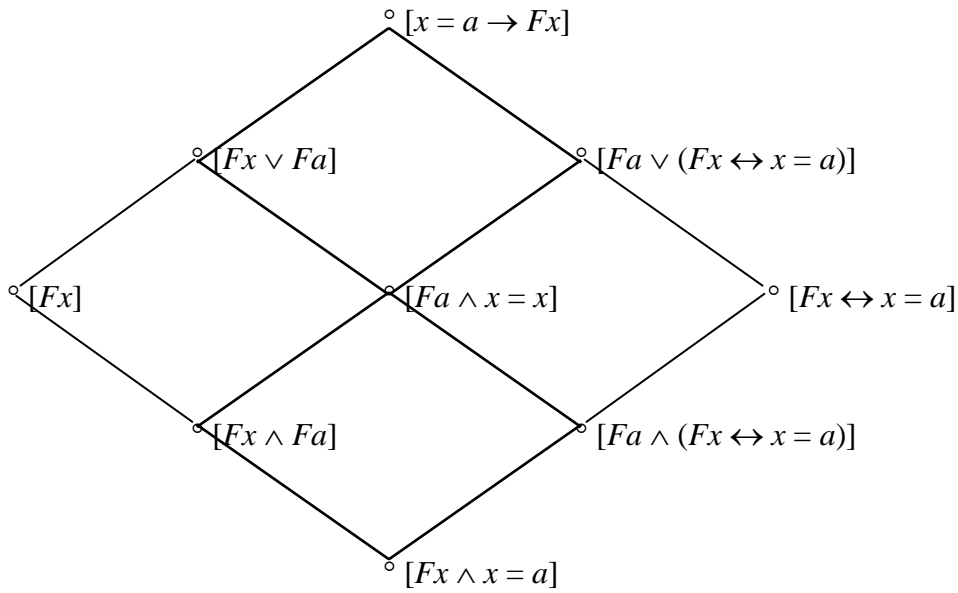


Figure 2.

The question naturally arises as to whether Figure 2 depicts all conditions $[\psi(x)]$ in $Cond(a, [Fa])$ with $\psi(x)$ constructed out of the non-logical vocabulary of Fa . A negative answer follows immediately from the fact that, when we take into account the possibility that $\psi(x)$ should contain quantifiers, we are confronted with infinitely many non-equivalent such $\psi(x)$, and therefore with infinitely many conditions $[\psi(x)]$ in $Cond(a, [Fa])$, even restricting ourselves to $\psi(x)$ whose sole non-logical vocabulary comprises F and a . The reason is as follows.¹⁰ Let E_n be the closed formula (built using quantifiers, variables, boolean connectives and ‘=’) saying that there are exactly n individuals ($n = 2, 3, 4, \dots$). Suitably selecting a pair of conditions from Figure 2—for example $[Fx]$ and $[Fx \leftrightarrow x = a]$ —we consider, for varying n , the condition $[(E_n \wedge Fx) \vee (\neg E_n \wedge (Fx \leftrightarrow x = a))]$. These conditions are distinct for distinct choices of n , and all belong to $Cond(a, [Fa])$, so there are denumerably many

¹⁰ I am indebted to Allen Hazen for this point, and for the way the formulas E_n are used here to make it.

$[\psi(x)] \in \text{Cond}(a, [Fa])$ with $\psi(x)$ free of non-logical vocabulary other than that contained in Fa .

What remains is the question—to which it will turn out that the answer is again negative—of whether Figure 2 gives a comprehensive picture of the set of $[\psi(x)] \in \text{Cond}(a, [Fa])$ for quantifier-free $\psi(x)$ built from F and a and the remaining logical vocabulary (remaining after the exclusion of quantifiers, that is). This amounts to considering such formulas $\psi(x)$ as can be constructed by boolean composition from the three atomic formulas Fx , Fa , and $x = a$, since we may ignore as vacuous $x = x$ and $a = a$ (and have already accounted for $a = x$, to within equivalence). By treating these three atomic formulas just listed as boolean atoms p , q , r (respectively), this reduces our question to a combinatorial problem in propositional logic.¹¹ There are 256 non-equivalent truth-functional compounds in three propositional variables, but we may ignore all of these which are inconsistent with the interpretation of r as $x = a$, which is to say, all which are inconsistent with the formula $r \rightarrow (p \leftrightarrow q)$. What remain are (to within equivalence) 64 compounds, whose disjunctive normal form representations are obtained by taking all possible disjunctions with disjuncts drawn from the set: $\{p \wedge q \wedge r, p \wedge q \wedge \neg r, p \wedge \neg q \wedge \neg r, \neg p \wedge q \wedge \neg r, \neg p \wedge \neg q \wedge r, \neg p \wedge \neg q \wedge \neg r\}$ (Here we have set aside the two ‘state-descriptions’ $p \wedge \neg q \wedge r$ and $\neg p \wedge q \wedge r$ as incompatible with $r \rightarrow (p \leftrightarrow q)$; since what remain are 6 such conjunctions, and we are to form all possible disjunctions, there are as many disjunctions as there are subsets of a 6-element set, i.e. $2^6 = 64$.) In this format, what corresponds to the substitution $x \mapsto a$ is the substitution according to which $p \mapsto q$ and $r \mapsto \top$ (\top being the constant-true formula).

An examination of cases reveals that precisely 16 of our 64 p,q,r -formulas are such that the composition of the latter pair of substitutions maps them to formulas (given $r \rightarrow (p \leftrightarrow q)$) equivalent to q . These 16 appear in Figure 3 below, with those which (or rather, whose corresponding conditions) appeared already in Figure 2 represented by solid rather than hollow dots in the diagram. Thus the answer to our question about the comprehensiveness of Figure 2 as a picture of the set of $[\psi(x)] \in \text{Cond}(a, [Fa])$ with $\psi(x)$ built from F , a , and logical vocabulary (quantifiers excepted) is that precisely seven such conditions are missing from the picture. We retain the p,q,r -representations in Figure 3 because the predicate-logical representations would take up too much space. Translating the seven conditions missing from Figure 2 back into the notation there gives:

$$\begin{aligned} & [Fx \leftrightarrow (Fa \vee x = a)], [(Fx \leftrightarrow Fa) \wedge (x = a \rightarrow Fx)], [(Fx \leftrightarrow x = a) \vee (Fa \leftrightarrow x = a)], \\ & [(Fx \vee Fa) \wedge ((Fx \wedge Fa) \rightarrow x = a)], [Fa \leftrightarrow x = a], [Fx \wedge (Fa \rightarrow x = a)], \\ & [(Fx \leftrightarrow x = a) \wedge (Fa \leftrightarrow x = a)], \end{aligned}$$

which are represented below, respectively, by $p \leftrightarrow (q \vee r)$, $(p \leftrightarrow q) \wedge (r \rightarrow p)$, and so on. We have used ‘+’ for exclusive disjunction.

¹¹ Here again I am much indebted to Allen Hazen.

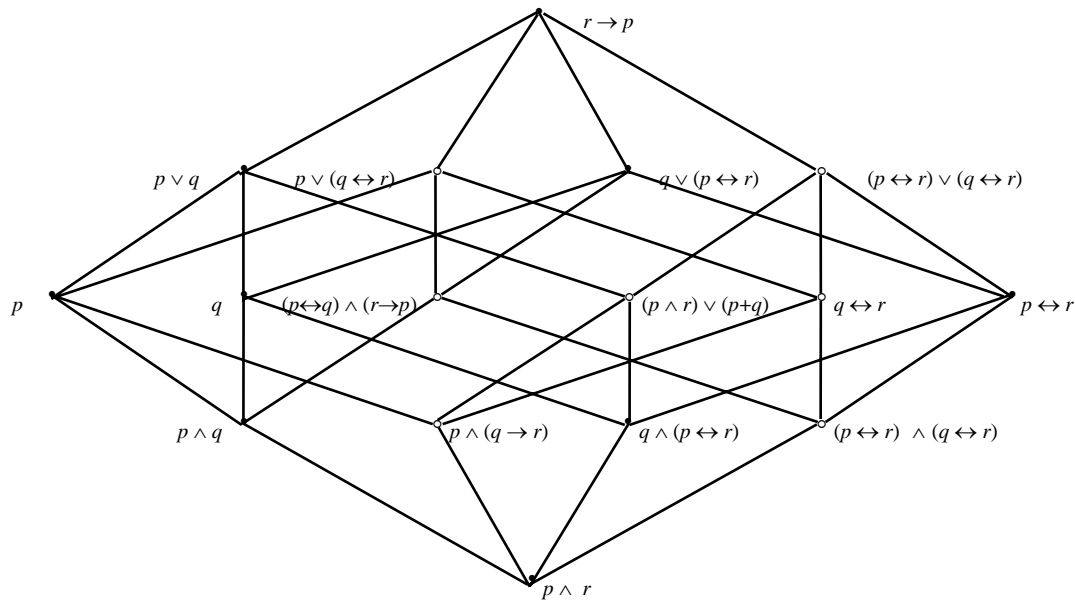


Figure 3.

Unlike Figure 2, Figure 3 depicts a boolean algebra, so that there is a unique complementation operation available; for the reasons given earlier, this complementation does not correspond (*à la* Lindenbaum–Tarski) to negation, and we need pay it no special attention. Like Figure 2, Figure 3 makes some arbitrary choices of representative formulas with which to label the nodes. For example, we could equally well have written $q \wedge r$, or $p \wedge q \wedge r$, in place of $p \wedge r$ at the bottom of the diagram; similarly, we could equally well have written $(p \vee q) \leftrightarrow r$ in place of $(p \leftrightarrow r) \wedge (q \leftrightarrow r)$. (The point so labelled is the lattice complement of $p \vee q$ and—something we shall just have to leave as a curiosity here—one always obtains the complement of a point labelled α in Figure 3 by taking that labelled by $\alpha \leftrightarrow r$ or some truth-functional equivalent thereof *modulo* the standing assumption $r \rightarrow (p \leftrightarrow q)$.) Finally, as in the case of Figures 1 and 2, matters would be considerably complicated here if we had dealt instead with the lattice of $[\psi(x)] \in \text{Cond}(a, [Raa])$ for quantifier-free $\psi(x)$ built from the non-logical vocabulary of Raa . Instead of looking at boolean compounds of three atomic formulas, we should have five to contend with (Rxx , Rax , Rxa , Raa , and $x = a$). The construction of a diagram analogous to Figure 3 for this case is best left to the (very) energetic reader.

Though a condition lattice like that in Figure 3 except based on a two-place rather than a one-place predicate letter is already too unwieldy to include in diagrammatic form here, let me close this section with one aspect of the binary case. In a dietitian's guide to food, Rosemary Stanton writes that human beings resemble sea creatures in that both retain salt within the body, and continues:

Humans also resemble at least some types of sea creature in that each likes to eat the other.¹²

Stanton would seem to have been misled by (surface) grammatical form into thinking that just as the phrase ‘retains salt within the body’ constitutes a predicate which applies both to human beings and (e.g.) sharks, so this is also the role played by the words ‘likes to eat the other’ (or ‘like to eat each other’) in the sentences concerned. To simplify discussion, let us change to an example in which it is proper names rather than predicates that picking out the (alleged) mutual resemblers: *a* and *b* resemble each other in that each bears the relation *R* to the other. (What Alf and Betty have in common is that each respects the other, for example.) It is true that from the hypothesis that each of *a* and *b* bears *R* to the other, there is a non-trivial condition both satisfy, namely: $[\exists y(Rxy \wedge Ryx)]$. (That is, the condition of bearing *R* to something that bears *R* to one.) But the statement that $Rab \wedge Rba$ is not the result of predicating this condition of either or each of *a*, *b*: after all each could bear the relation *R* to something that bears the relation *R* back to it without their bearing *R* to each other. (Cf. note 3.) Can one nonetheless find a condition whose satisfaction by *a* amounts to the truth of $Rab \wedge Rba$, and whose satisfaction by *b* also amounts to the truth of $Rab \wedge Rba$? One such condition would be:

$$(0) \quad [(x = a \rightarrow Rxb) \wedge (x = b \rightarrow Rax)]$$

For on replacing the ‘*x*’ by ‘*a*’ the first conjunct of (0) becomes (equivalent to) Rab , and this implies the result of making the same replacement in the second conjunct; thus the conjunction is equivalent to Rab . (*Mutatis mutandis* for the case of *b*.) This does not, however, seem a very promising basis for claiming *a* and *b*’s *R*-ing each other as a respect of resemblance, since in each case we can change the second occurrence of *R* to *S* and obtain something which when predicated of *a* or *b* gives $Rab \wedge Sba$, subject to the additional assumption—arguably implicit in the ‘other’ of ‘each other’—that $a \neq b$. (See our discussion of $a \neq b$ -equivalence in Section 4 below.) Thus *a* and *b* would have to be said to resemble each other in virtue of satisfying the modified condition in the situation in which *a* respects *b* while *b* despises *a*. We leave the reader to explore further variations on the theme of (0), to see if anything more promising might be found as a way of construing the claim that Rab and Rba as the attribution of a common property to *a* and *b* without yielding an analogous construal even when the second ‘*R*’ is replaced by ‘*S*’. Consideration could be made here also of conditions specified (unlike (0)) by open formulas with internal quantificational complexity; we will not investigate the matter here, however.

4. The Doctrine of Principal Attributes

Fitch’s response, quoted in Section 2 above, to the fact—apparently posing a difficulty for the $a \setminus \varphi$ notation purporting to denote a single condition—that the set we have been calling $Cond(a, [\varphi])$ typically contains more than one element, is to take the former notation to denote a particular element of that set, the so-called principal attribute

¹² Rosemary Stanton’s *Complete Book of Food and Nutrition*, Simon and Schuster (Australia), East Roseville, N.S.W. 1989. The quoted passage appears on p.306.

assigned to a by φ . Further, it seems clear from the quoted passage that this proposal amounts to what in our introductory section was called the ‘syntactical construal’ of $a \setminus \varphi$. It remains to be seen whether the individuation policy assumed here for propositions (‘identity by equivalence’, in a nutshell) is consistent with the proposal.

The proposal under consideration is that from amongst all the elements of $Cond(a, [\varphi])$, which is to say all those conditions $[\psi(x)]$ for which we have $\psi(x \mapsto a) \dashv\vdash \varphi$, we can single out a distinguished element (or ‘principal condition’, for ‘ $a \setminus \varphi$ ’ to denote) by insisting that $\psi(x)$ does not contain a .¹³ The proposal would be inconsistent if the following possibility were realized: we could find $\psi_1(x)$, $\psi_2(x)$, neither containing the constant a , each yielding a formula equivalent to (some given) φ on substitution of a for x , and yet determining distinct elements of $Cond(a, [\varphi])$. That is, despite the equivalence just mentioned, for the open formulas concerned, we should have a failure of the equivalence $\psi_1(x) \dashv\vdash \psi_2(x)$. This supposed possibility is not, however, a genuine possibility after all, since from the fact that $\psi_1(x \mapsto a) \dashv\vdash \varphi$ and $\psi_2(x \mapsto a) \dashv\vdash \varphi$ we infer that $\psi_1(x \mapsto a) \dashv\vdash \psi_2(x \mapsto a)$, and *ex hypothesi* neither $\psi_1(x)$ nor $\psi_2(x)$ contains a : the substitution $x \mapsto a$ is therefore invertible, and when we apply $a \mapsto x$ to both sides of this last equivalence, we obtain the needed equivalence $\psi_1(x) \dashv\vdash \psi_2(x)$.

Are we to conclude that the doctrine of principal attributes does not after all depend on an arbitrary attachment to this rather than that mode of linguistic expression? What of the argument given in Section 1? We there considered a monadic predicate G , stipulated to hold of an object just in case that object bears the relation R to a , and claimed that (what we are now following Fitch in calling) the principal attribute $[Gx]$, alias $[Rxa]$, assigned to a by $[Ga]$ was different from the principal attribute, $[Rxx]$, assigned to a by $[Raa]$: this purports to be a *Reductio* of the idea of principal attributes since $[Ga] = [Raa]$. What stands in need of clarification here is the idea of a ‘stipulation’. The argument of the preceding paragraph used the relation $\dashv\vdash$ of narrowly (‘formally’) logical equivalence, more specifically with the underlying logic taken to be classical predicate logic: but no amount of stipulation will make the distinct atomic formulas Ga and Raa (or Gb and Rba) logically equivalent in this sense! What about the idea of defining ‘ G ’ by putting Gx as *definiendum* and Rxa as *definiens*? To address this question, we need to recall the distinction between the meta-linguistic and the object-linguistic conceptions of definition—the distinction Meyer (1974) puts as that between ‘dishonest’ and ‘honest’ approaches to definition (in which discussion, admittedly, it is definitions of the logical vocabulary, rather than, as here, the non-logical vocabulary, that are at issue). On the former approach, the defined vocabulary serves merely to abbreviate, in the meta-language, reference to the vocabulary of the object language which is not itself increased by the procedure of definition. On the latter approach, the object language’s vocabulary is itself enriched by the addition of the defined expressions. If we stick to the metalinguistic construal, then the informal argument above errs in taking $[Rxa]$ as the principal attribute assigned to a by $[Ga]$, when Fitch’s syntactical procedure is followed, since the expression we refer to by ‘ Ga ’ is none other than the

¹³ Note that we are not saying that $[\psi(x)]$, for a -free $\psi(x)$, may not also be $[\psi'(x)]$ where $\psi'(x)$ does contain a ; in this case of course the occurrences of a in the latter formula are what are called ‘inessential’ occurrences, in the sense that formula is logically equivalent to one (*viz.* $\psi(x)$) in which there are no such occurrences.

object language sentence *Raa*, operating on which, that procedure—replacing all occurrences of ‘*a*’ by ‘*x*’—delivers [*Rxx*], not [*Rxa*]. It remains to be seen next what happens if we take instead the object-linguistic approach.

On the simplest implementation of the latter approach, we have in mind some first-order theory whose non-logical vocabulary includes *R* (and *a*) but not *G*, and we simultaneously enrich the language by adding *G*, and extend the theory with the aid of an explicit definition: $\forall x(Gx \leftrightarrow Rxa)$.¹⁴ Since the latter has the status of a non-logical axiom, the conclusion of the proof two paragraphs back (to the effect that $\psi_1(x) \dashv\vdash \psi_2(x)$) stands, because, to repeat, the ‘ $\dashv\vdash$ ’ represents logical equivalence, not equivalence *modulo* some set of non-logical axioms. A slightly more complex version of the object-linguistic approach would give such a definition a status distinguishing it from an arbitrary non-logical axiom – the status of a meaning postulate (Carnap (1947), Appendix B). Especially in the context of an empirical theory, this special status is intended to mark the biconditional in question as not making an empirical claim while at the same time conceding that it does not owe its *a priori* character to strictly formal logical considerations. One can think of such non-formal analyticities as a way of overcoming the atomism built into any approach recognising only the narrowly logical relations between complex predicates. The atomism here complained of is manifested in the fact that such complex predicates are composed ultimately from atomic predicates, all (non-logical) such predicates being logically independent. Meaning postulates are intended to cancel this independence out by registering some *a priori* (if not ‘narrowly’ logical) meaning relations amongst the distinct atomic predicates of an interpreted language. In the case of a theory whose intended interpretation is such that all of its theorems aspire to the status of truths knowable *a priori*, we can perhaps dispense with the need to segregate off a proper subset of the axioms as meaning postulates; at any rate, it is in the context of such a theory that we shall attempt to undermine the doctrine of principal attributes.

The theory we have in mind is a fragment of the arithmetic of integers, with as its primitive non-logical vocabulary the individual constant 0 (‘zero’), the two-place predicate symbol $>$ (‘greater than’), the one-place predicate symbol ‘*P*’ (‘positive’), and the one-place function symbol ‘*s*’, for the successor function, so that we can form names for all the natural numbers. As usual we shall abbreviate $s(s(\dots(s(0))\dots))$, where there are *n* occurrences of ‘*s*’, to ‘*n*’. (Such terms are called numerals.) The theorems of the theory are to be all truths formulable in the first-order language based on this vocabulary, under its intended interpretation (as given by the parenthetical glosses) with the quantifiers ranging over integers. These include of course

$$\forall x(Px \leftrightarrow x > 0)$$

showing that, though here we take it as a primitive symbol, *P* is in fact explicitly definable in the theory. Now where *n* is a numeral, what should we say about the propositions expressed by the two sentences $n > 0$ and Pn ? Whether or not the biconditional inset above is given a special ‘meaning postulate’ status, a case can be made for saying that its analyticity should make us count the proposition expressed by $n > 0$ and that expressed by Pn (for a given choice of *n*) as the same proposition, the failure of the two sentences to count as (‘narrowly’) logically equivalent

¹⁴ A justly famous treatment of this implementation may be found in Chapter 8 of Suppes (1957).

notwithstanding.¹⁵ Let us take, in particular, the case in which n is 0. What, to invoke Fitch's notation, is $0 \setminus P0$, the principal attribute ascribed to 0 by $P0$? It is supposed to be the property of being a positive integer, $[Px]$ in our notation ($'\lambda x.Px'$, to use the lambda notation), since this is what we obtain from $P0$ by replacing all occurrences of '0' by the variable x . And what is $0 \setminus 0 > 0$, the principal attribute ascribed to 0 by $0 > 0$? Following the same procedure, we arrive at the property of being greater than oneself, $[x > x]$ (or $\lambda x.x > x$). But—a point already familiar from the passage quoted from Salmon (1986) in Section 2 above—the first of these is a property possessed by many numbers, while the second is one possessed by none. So, on the assumption that we can regard any pair Pn and $n > 0$ as expressing the same proposition, there is simply no well-defined operation from propositions to properties specified by the syntactical procedure of abstracting the principal attribute from a sentence expressing the proposition in question: the result of that procedure is inappropriately sensitive to the chosen representative from amongst the various sentences expressing a given proposition. Thus everything hangs on whether or not the above assumption about the individuation of propositions is accepted. In its favour, let me say just this. Once we have decided to adopt a coarse-grained individuation policy to the extent of allowing (narrowly) logically equivalent sentences of an interpreted language to express the same propositions, it would be artificial to stop at this point rather than going on and extending the same privilege to *a priori* equivalent sentences. And as we have seen, the doctrine of principal attributes does not withstand this natural extension.

A popular coarse-grained individuation procedure for propositions consists in identifying them with sets of possible worlds, the proposition expressed by a given (non-indexical and unambiguous) sentence comprising precisely the worlds at which that sentence is true. No-one who accepts the distinction between necessity and *a priori* knowability would be happy with this proposal, however, at least if propositions are to be thought of as the objects of the propositional attitudes of an ideally rational and reflective subject. There are ways of complicating the set-of-possible-worlds account which afford a reasonable approximation to what one wants from propositions (i.e., that *a priori* equivalent sentences should express the same proposition)—for example, as sets of (ordered) pairs of worlds—but it would take us too far afield to discuss the relevant issues here.¹⁶ On either the simple minded set-of-worlds account or any such refinement, well known cardinality considerations give us propositions not expressed by any sentence of the interpreted language under discussion. The less ambitious 'algebraic

¹⁵ If it is felt to detract from the example that all sentences of the theory are *a priori* equivalent (since all are knowable *a priori*) and so express the same proposition, one can modify the case so that the arithmetical theory is incorporated into a broader empirical theory.

¹⁶ The idea is that for $\langle w_1, w_2 \rangle$ to belong to the proposition expressed by a sentence ϕ is for ϕ , considered as uttered in w_1 , to be true of w_2 . See Davies and Humberstone (1980) for a defence of such semantic values for (modally) non-indexical sentences and the connection with *a priori* knowability; the relevant ideas go back to work in the 1970's by Kaplan, Stalnaker and van Fraassen, references to which may be found in Davies and Humberstone. In deference to the set-of-worlds notion of a proposition, workers in this paradigm ('two-dimensional modal logic') have usually felt it desirable to use a different term for such semantic values; they (or some equivalent) have been called 'dipositions' and 'propositional concepts', for example, by different authors. Sameness of semantic value, under whatever name, for a pair of sentences ϕ, ψ , is a stronger property than the *a priori* knowability of the biconditional $\phi \leftrightarrow \psi$, though the difference arises only in connection with modal embeddings not under consideration here. (In the terminology of Davies and Humberstone, the stronger property is recorded by prefixing the biconditional with 'fixedly necessarily', and the weaker by prefixing with 'fixedly actually'.)

logic'-inspired account that we have been working with, on which propositions are identified with equivalence classes of sentences under logical equivalence, as well as the refinement we have just suggested, with '*a priori* equivalence' replacing 'logical equivalence', does not provide a place—though it could be extended to do so—for such inexpressible propositions. However, if a condition is taken to be any function from individuals to such propositions, then room has still been made available for inexpressible conditions: that is, conditions which are not of the form $[\psi(x)]$ for any open formula $\psi(x)$ of the language under consideration.¹⁷ (The expressible conditions $[\psi(x)]$ themselves count as conditions on this functional definition, with the further simplifying assumption in force that every individual has a name, since such a condition maps an individual *a* to the proposition $[\psi(x \mapsto a)]$.¹⁸) We return to the familiar expressible conditions for the remainder of the discussion.

A glance at Figure 3 with the 'key' in mind that *p*, *q*, and *r* stand for *Fx*, *Fa*, and $x = a$, respectively, will reveal that when thus decrypted, the only node labelled with a condition specified without the aid of any occurrences of the name *a* is that on the left of the central tier, whose label is *p*. Does this give the Fitchian principal condition [*Fx*] thereby represented a special status in the lattice, our earlier (§2) remarks to the contrary on this score notwithstanding? Not once logical equivalence is replaced by *a priori* equivalence, as illustrated by the arithmetical example above. An illustration bearing more directly on the present line of thought may be given thus. Suppose that we have two atomic predicates *F* and *G* with the following relationship between them (and *a*) assumed to hold *a priori*:

$$(1) \quad \forall x(Gx \leftrightarrow (Fx \leftrightarrow x = a))$$

On the current individuation policy for propositions (and therefore for conditions), we can relabel the nodes with new labels based on *Gx*, *Ga*, and $x = a$. For instead of '*Fx*' (or more accurately '*[Fx]*') we write ' $Gx \leftrightarrow x = a$ ', and replacing '*Fa*' we have '*Ga*'.¹⁹ Since the new open formulas are *a priori* coextensive with those they supplant, only the labels for the conditions have been changed, not the conditions themselves. Now, taking

¹⁷ This follows again in view of cardinality considerations, assuming the language has infinitely many (pairwise non-coreferential) names.

¹⁸ This connection with functions will remind readers of the similarity between asking what attribute a particular proposition assigns to a given object and asking what function yields a particular value for a given argument. As Sullivan (1992), p.93 says *à propos* of the latter question—echoing a sentiment Dummett has emphasized in Fregean exegesis—"Specification of its value for a single argument signally fails to specify a function". I have not pressed this analogy here because it seems to do insufficient justice to the degree to which the doctrine of principal attributes is successful (i.e., when narrowly logical equivalence is employed, as we saw earlier in this section.)

¹⁹ We do not have to replace ' $x = a$ ', of course, since we are concerned only to eliminate *F* in favour of *G*. The replacement of '*Fx*' by ' $Gx \leftrightarrow x = a$ ' is justified by the (narrowly logical!) equivalence between (1) and:

$$\forall x(Fx \leftrightarrow (Gx \leftrightarrow x = a)).$$

Our procedure here will seem more than vaguely reminiscent of Goodman's *grue* example (for which the reformulation of (1) as (2) below may be more suggestive) and Miller's Arizona-Minnesotan example, from two classic papers in the genre of exposing inappropriately language-dependent analyses: Goodman (1955), Miller (1974). In their cases, the contrast was between equivalent formulations in different (intertranslatable) languages, rather than, as here, equivalent formulations in the same language; the present example could be recast as a 'two languages' example, though to avoid an excursus into questions of cross-language logical relations (*cf.* Humberstone (1988), p.402) I have not done so here.

Figure 2 rather than the more cluttered Figure 3, its central tier of conditions, $[Fx]$, $[Fa \wedge x = x]$, $[Fx \leftrightarrow x = a]$, can be redescribed as $[Gx \leftrightarrow x = a]$, $[Ga \wedge x = x]$, and $[Gx]$, and this time it is the third rather than the first that is labelled without recourse to the name a . Will the real principal attribute please stand up? The point of course is understated in saying that the principal attribute assigned to a by $[Fa]$ occupies no distinguished position in the lattice $Cond(a, [Fa])$ —though that formulation was satisfactory when only narrowly logical equivalence was employed for the individuation of propositions: rather, there simply is no principal attribute about whose lattice position to enquire. If we apply the syntactical procedure to the sentence Fa , we get $[Fx]$, while if we apply it to Ga we get $[Gx]$: but these are nonequivalent conditions whereas *ex hypothesi* $[Fa]$ and $[Ga]$ were the same proposition.

A variation on (1), above, can usefully be brought to bear on a topic which van Fraassen (1982) calls by that very name, ‘variation’. For this purpose we do best to consider an equivalent formulation of (1):

$$(2) \quad \forall x(Gx \leftrightarrow ((x = a \wedge Fx) \vee (x \neq a \wedge \neg Fx)))$$

The variation we have in mind selects another name and another atomic predicate (b and F' respectively):

$$(3) \quad \forall x(Gx \leftrightarrow ((x = a \wedge Fx) \vee (x = b \wedge F'x)))$$

For what van Fraassen means by ‘variation’ we excerpt some of the informal remarks from pp.343–5 of his 1982 paper.²⁰

Contemplating the proposition that Socrates is mortal, we can abstract the property of (someone’s) being mortal, and generalize to produce the proposition that everyone has this property. (...) How does abstraction proceed? In one model of the process, the contemplated proposition is seen as complex, as having distinct ingredients (...) and abstraction is a sort of deletion or separation, (...) A second model of the process of abstraction utilizes the ideas of mental variation and invariance. Suppose I transform the initial proposition successively into: that Callias is mortal, that Gorgias is mortal, ... The abstract (i.e. what is abstracted, what is ‘common’ to all these propositions) can equally well be represented by the set of all these resultant propositions themselves. (...) Names (...) appear simply as labels for abstractors. (...) “Tom” is the label of the abstractor which, intuitively speaking, abstracts the property being tall from the proposition that Tom is tall. Two such abstractors may be correlated; intuitively, what the Tom abstractor does to that Tom is tall is just what the Harry abstractor does to that Harry is tall. To put it slightly more precisely, there is a Harry-for-Tom operation, which turns that Tom is tall into that Harry is tall, and this operation (I call it, naturally, a variation) allows us to express the relation between these two abstractors.

Although these remarks are prefatory to van Fraassen’s formal development (which it is no part of my intention to criticize) of a theory of propositions, abstractors and variations, it is perhaps permissible to look at what, more than once, are presented as intuitive underpinnings without attending to that elaborate superstructure. Several points

²⁰ More in this vein may also be found in van Fraassen (1988).

in the above quotation may make us wonder about the motivating ideas. The Tom abstractor purports to do what we have seen reason to believe nothing can do—take us from a name (or its bearer) and a proposition to the property the proposition ascribes to the bearer—unless perhaps the formal theory construes the key notions here in specialized ways.²¹ Suspicions on this score are increased by such later passages (p.347) as the following:

In Fitch’s perspicuous symbolism, $[a\Fa]$ is the property that a must have in order for Fa to be true, and $U[a\Fa]$ the assertion that this property is universal: everything has it. Clearly $[b\Fb] = [a\Fa]$ and $U[b\Fb] = U[a\Fa]$. The reason is that Fa and Fb are two propositions which are ‘congruent’ in a certain sense; each can be turned into the other by a simple variation.²²

Fitch’s slick symbolism, we have been arguing, is anything but perspicuous: once propositions—at least, as currently individuated—are distinguished carefully from sentences, we find that there is nothing answering to this seductive notation. Nor, a propos of the ‘U’ in this passage, is there such a thing as the property whose universality is claimed by the proposition expressed by a universally quantified sentence. (The proposition that everyone likes everyone is true just in case the property of liking everyone is universal, as well as just in case the—quite different—property of being liked by everyone is universal, for example.)

What remains of van Fraassen’s idea of variation, of (e.g.) the “*Harry-for-Tom* operation, which turns *that Tom is tall* into *that Harry is tall*”, once we abandon the doctrine of principal attributes and the associated privileging of a single element of the set of conditions or attributes we would call $Cond(Tom, [Tom\ is\ tall])$? For each element, $[\psi(x)]$, of this set, there is the proposition $[\psi(Harry)]$. The proposition that Harry is tall is merely one among many elements of this set. Can we perhaps show that for any two propositions $[\phi_1]$ and $[\phi_2]$ and any individuals a, b , there is some condition $[\psi(x)]$ in $Cond(a, [\phi_1]) \cap Cond(b, [\phi_2])$? To do so would be to show that the proposition that Tom is tall and the proposition that Harry is tall are not brought into any especially intimate relationship by the observation that the one attributes a property to Tom which the other attributes to Harry: for the same would apply even if—keeping the first as it is—the second had been the proposition that Harry is Russian, or indeed the proposition that Marilyn Monroe owned a cat.

What is wanted, then, is a way of constructing $\psi(x)$ from ϕ_1, ϕ_2 , for which we have $\psi(x \mapsto a) \dashv\vdash \phi_1$ and $\psi(x \mapsto b) \dashv\vdash \phi_2$. This will put $[\psi(x)]$ into $Cond(a, [\phi_1]) \cap Cond(b, [\phi_2])$, as desired. Taking our inspiration from (3) above, we might suggest putting $\psi(x) =$

$$(4) \quad (\phi_1 \wedge x = a) \vee (\phi_2 \wedge x = b)$$

²¹ In particular, a more fine-grained criterion than that employed here for individuating propositions would need to be supplied. Since van Fraassen favours what he calls, in the passage quoted above, the second model (‘propositions as black boxes’, as he also puts it), this means we should distinguish ‘fine-grained’ from ‘quasi-syntactic’ accounts of propositional identity: the latter label seems to suggest instead the first model. Unfortunately, there is little in van Fraassen (1982) or (1988) to tell us which sentences express which propositions — in particular, on the matter of when distinct sentences express the same proposition.

²² In reproducing this passage I have replaced slashes by backslashes, since van Fraassen (who actually writes ‘ $[a\Fa]$ ’, etc.) evidently intends to be employing Fitch’s notation.

This gives $\psi(x \mapsto a)$ logically equivalent to

$$(5) \quad \varphi_1 \vee (\varphi_2 \wedge x = b)$$

Let us say that formulas φ and φ' are $(a \neq b)$ -equivalent when they are logically equivalent given the additional assumption that $a \neq b$.²³ Then, since this additional assumption ‘knocks out’ the second disjunct of (5), (4) is $(a \neq b)$ -equivalent to φ_1 . Similarly, $\psi(x \mapsto b)$ is $(a \neq b)$ -equivalent to φ_2 . If φ_1 and φ_2 happen to contain a and b , for example, because they are respectively Fa and Gb , in which case (4) becomes

$$(6) \quad (Fa \wedge x = a) \vee (Gb \wedge x = b)$$

then various alternative formulations are available, such as

$$(7) \quad (Fx \wedge x = a) \vee (Gx \wedge x = b)$$

and if there are several occurrences of a in φ_1 or b in φ_2 , one has the choice of supplanting some or all of them by ‘ x ’. (Because (6) and (7) are disjunctwise logically equivalent, we have alternative formulations here, rather than different conditions.)

Where the non-identity $a \neq b$ is *a priori* knowable, $(a \neq b)$ -equivalent sentences express the same proposition and $(a \neq b)$ -equivalent open formulas determine the same condition, if the considerations aired above are at all persuasive. This would appear to be the status, not only of such non-identities as the arithmetical $0 \neq 3$, but also where a and b are respectively Marilyn Monroe and the Eiffel Tower. Thus, following the style of (7), the propositions expressed by ‘Marilyn Monroe owns a cat’ (φ_1) and ‘The Eiffel Tower contains no zinc’ (φ_2) can be thought of as attributing (to a and to b) the property of either owning a cat and being Marilyn Monroe or containing no zinc and being the Eiffel Tower.

Choosing $\psi(x)$ for a given φ_1 , φ_2 , a , and b , as (4) does, makes $[\psi(x)]$ a hard condition to satisfy: only a and b have a chance of doing so. But this is not so for all elements of $Cond(a, [\varphi_1]) \cap Cond(b, [\varphi_2])$. To go to the opposite extreme, we could consider (8):

$$(8) \quad (x = a \rightarrow \varphi_1) \wedge (x = b \rightarrow \varphi_2)$$

The reader is invited to check that $x \mapsto a$ and $x \mapsto b$ operate on (8) to give $(a \neq b)$ -equivalents of φ_1 and φ_2 , respectively.²⁴ This time the condition is especially easy to

²³ I.e., when $\varphi, a \neq b \vdash \varphi'$ and $\varphi', a \neq b \vdash \varphi$.

²⁴ In view of the similarity between the condition labelled (0) at the end of Section 3, where it was equivalence *simpliciter* rather than the arguably somewhat artificial notion of $(a \neq b)$ -equivalence that was involved, whether we cannot follow the lead of that discussion and eliminate the latter notion. Take the case in which φ_1 and φ_2 are Fa and Gb respectively. (Here ‘ a ’, ‘ b ’ function temporarily as variables because we do not want to invite confusion with the variable ‘ x ’ in terms of which the condition we are leading up will be stated.) This makes R what has been called (*cf.* Humberstone (1996), p.219) an ‘ \wedge -representable’ relation: intuitively none too relational a relation—since its holding between a and b is simply a matter of a ’s and b ’s each meeting conditions specifiable without reference to the other—but one we use to copy the form of our earlier (0), to arrive at the condition:

$$[(x = a \rightarrow (Fx \wedge Gb)) \wedge (x = b \rightarrow (Fa \wedge Gx))]$$

We can follow out the results of applying this condition to a and to b in turn, as in Section 3 we did for (0), without invoking the notion of $(a \neq b)$ -equivalence; but the result is that we obtain, as a result of each application the same proposition, namely $[Fa \wedge Gb]$: whereas what was wanted was, for the one application (i.e., for the substitution $x \mapsto a$) $[Fa]$ and for the other $[Gb]$. Recourse to $(a \neq b)$ -equivalence can be seen to be required by the following consideration: if $[\varphi_1]$ and $[\varphi_2]$ are distinct propositions, there

satisfy, in that it is automatically satisfied by everything other than *a* and *b*, and is satisfied by them according as φ_1 or φ_2 (respectively) is true. Again reformulations may be available in the style of (7) here. With φ_1 and φ_2 , *a* and *b* as at the end of the preceding paragraph the common condition in this style attributed to the individuals concerned is—to use rather strained English—that of being such that if one is Marilyn Monroe one owns a cat and if one is the Eiffel Tower one contains no zinc. While the disjuncts of (4) are represented at the bottom of Figure 3 in the preceding section, the conjuncts of (7) appear at the top; the labels on intermediate nodes in that diagram will suggest still further distinct conditions each of which a pair of proposition can be regarded as imposing on a pair of individuals. To the extent to which our coarse-grained way with propositions is found appealing, we conclude, the prospects for an interesting notion of van Fraassen-style variation are no rosier than those for the notion of abstraction.²⁵

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cannot be a function (“the result of applying the condition $[\psi(x)]$ to”) which yields them as the respective values of that function for arguments *a*, *b*, when *a* = *b*. In spite of this obvious *a priori* consideration, the author was at one time tempted into thinking that following the line of (0) the appeal to (*a* ≠ *b*)-equivalence could be avoided, and this note is included to spare the reader the same confusion.

²⁵ I have been greatly assisted in preparing this material by comments from Bas van Fraassen, Allen Hazen, Richard Holton and Thomas Bull. The paper was originally delivered at the Australasian Association for Logic annual conference in Brisbane, July 1996.

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